Magnetic field effects in the heat flow of charged fluids.

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Abstract

Heat conduction in ionized plasmas in the presence of magnetic fields is today a fashionable problem. The kinetic theory of plasmas, in the context of non-equilibrium thermodynamics, predicts a Hall-effect-like heat flow due to the presence of a magnetic field in ionized gases. This cross effect, the Righi-Leduc effect together with a heat flow perpendicular to the magnetic field are shown to yield significant contributions, under certain conditions to the ordinary Fourier component of the heat flow. The thermal conductivities associated with these effects change with the strength of the magnetic field for a given temperature and density and are shown to be significant compared with the parallel conductivity for a whole range of values of $\vec{B}$.

1 Introduction

The behavior of charged particles in the presence of external magnetic fields is a well known subject. According to the tenets of classical non-equilibrium thermodynamics for not too large gradients, the simultaneous heat and electrical flows in a magnetic field are linear functions of the temperature and electrical

potential gradients, respectively. The coefficients appearing in these relations are, in general, second rank tensors which are functions of the external field and consist of a symmetrical part and an antisymmetric Al part [1, 2, 3]. The latter ones are called the “Hall vector” in the case of electrical conduction and the “Righi-Leduc” vector in the case of heat conduction. The former is a well known effect whereas the latter one isn't. It was discovered in 1887 and first measured and confirmed by Waldemar Voigt in 1903 (see Refs. [2, 3]). Curiously enough it has been, ever since, hardly mentioned in the literature. Not even the authors of a rather beautiful and striking experiment published recently [4] recognize that what they have really detected is the Righi-Leduc effect.

In this paper we address ourselves to a rather different calculation of the heat conduction effects in magnetized plasmas from the theoretical point of view. Indeed, in the case of a dilute ionized gas in the presence of weak magnetic fields for densities n in the interval $10^3 \leq n \leq 10^8\, m^{-3}$ and temperatures in the range $10^3 < T < 10^7\, K$ the magnitude of heat, charge and mass currents arising from these and other so-called “cross effects” may contribute by factors which are not necessarily negligible when compared with those arising from ordinary heat, mass and electrical conduction. This result seems to differ from those obtained by Braginski [5] while qualitatively agree with those obtained by Balescu in his excellent and exhaustive treatment on plasma transport processes [6]. We shall come back to this point a little later in this paper. Moreover, we believe that the approach here offered to the general subject of collisional transport processes in plasmas may provide a better understanding of this subject at temperatures and densities where collisions are favored in a non-relativistic framework.

The structure of the paper is as follows. In section II we shall briefly review the kinetic concepts used throughout the work, section III contains the bulk of the main calculations leading to the heat conduction transport coefficients and in section IV the results are given together with some concluding remarks.

2 Kinetic model

The kinetic model we use in our approach to the problem is the well known one based on the Boltzmann equation as originally proposed by Chapman and Cowling [7]. The substantial difference is that for the case of ionized gases they never developed their method up to the stage of placing it within the framework of linear irreversible thermodynamics. Although they derive elementary expressions for the Righi-Leduc and the Nernst-Ettingshausen effects (see Ref. [2]) they never pursued any comparison of their formulae with experiments, much less studied any possible applications. Here we would like to focus ourselves on the former and related effects.

Thus, the system here considered is a binary mixture of electrically charged particles with masses and charges m_i and e_i for $i = a, b$. The density of the system is low enough such that the kinetic description is valid. For simplicity we shall set the charge of the ions $Z = 1$. For such a system, the evolution of

the distribution functions of the molecules is given by the Boltzmann equation:

$$\frac{df_i}{dt} = \frac{\partial f_i}{\partial t} + \vec{v_i} \cdot \frac{\partial f_i}{\partial \vec{r}} + \frac{e}{m_i}\left(\vec{E} + \vec{v_i} \times \vec{B}\right) \cdot \frac{\partial f_i}{\partial \vec{v_i}} = \sum_{i,j=a}^{b} J\left(f_i f_j\right) \tag{1}$$

where the subscript i indicates the species and $J\left(f_i f_j\right)$ is the collisional term representing collisions between different and same species. The Lorentz force on the left side of Eq. (1) is treated here as an external force where the magnitude of the magnetic field is small enough such that collisions dominate over cyclotron motion. We must clarify that both the electric and magnetic fields appearing in this force contain the self consistent fields produced by the plasma and are governed by the Maxwell's equations (see Ref. [6]). This weak field approximation implies $\omega_i \tau \approx 1$ where $\omega_i = eB/m_i$ is the Larmor frequency, that is, the frequency of the circular orbits that the particles describe around magnetic field lines.

Once Eq. (1) is defined, the derivation of the conservation equations as well as the proof of the H-theorem are logically required. The former is a standard step widely discussed in the literature ([6]-[10]). The second result becomes a little bit more tricky due to the presence of a magnetic field but we will not bother with it since it is not essential to this paper [8]. We here proceed directly with the solution of Eq. (1) following the standard Hilbert-Chapman-Enskog approximation. Since a local Maxwellian distribution function $f_i^{(0)}$ is clearly a solution to the homogeneous part of Eq. (1) we propose that the single particle distribution functions f_i $(i = a, b)$ may be expanded around $f_i^{(0)}$ in a power series of Knudsen's parameter ϵ which, as well known, is a measure of the magnitude of the macroscopic gradients [7, 9, 10]. Thus

$$f_i = f_i^{(0)}\left[1 + \epsilon\varphi_i^{(1)} + \mathcal{O}\left(\epsilon^2\right)\right] \tag{2}$$

In Eq. (2) the functional equilibrium assumption is also invoked, namely the time dependence of $f_i^{(0)}$ and $\varphi_i^{(n)}$ for all n occurs only through the conserved densities. The particle density is $n_i\left(\vec{r}, t\right)$ $(i = a, b)$, the barycentric velocity $\vec{u}\left(\vec{r}, t\right)$ and the local temperature $T\left(\vec{r}, t\right)$ is in this paper assumed to be the same for ions and electrons [11]. $T\left(\vec{r}, t\right)$ is related to the internal energy density $\varepsilon\left(\vec{r}, t\right)$ by the standard ideal gas relationship. In this work we shall deal only with the first order in the gradients correction to f_i characterized by $\varphi_i^{(1)}$, namely the Navier-Stokes-Fourier regime.

Substitution of Eq. (2) into Eq. (1), leads to order zero in ϵ to the Euler equations of magnetohydrodynamics. To first order in ϵ one obtains a set of two linear integral equations for $\varphi_i^{(1)}$ which involve the linearized collision kernels in their homogeneous terms whereas the inhomogeneous ones contain a combination of terms involving the macroscopic gradients ∇T, $\nabla \vec{u}$ and the diffusive force $\vec{d_{ij}} = -\vec{d_{ji}}$. Indeed, after a somewhat lengthy but standard manipulation

[7] one gets that

$$\frac{m_i}{kT}\overset{\circ}{\vec{c}_i \vec{c}_i}:\nabla\vec{u}+\left[\left(\frac{m_i c_i^2}{2kT}-\frac{5}{2}\right)\frac{\nabla T}{T}+\frac{n_i}{n}\vec{d}_{ij}\right]\cdot\vec{c}_i=-\frac{m_i}{\rho kT}\sum_{j=a}^{b}e_j\int d\vec{c}_j f_j^{(0)}\varphi_j^{(1)}\left(\vec{c}_j\times\vec{B}\right)\cdot\vec{c}_i$$
$$-\frac{e_i}{m_i}\vec{c}_i\times\vec{B}\frac{\partial\varphi_i^{(1)}}{\partial\vec{v}_i}+\mathrm{C}\left(\varphi_i^{(1)}\right)+\mathrm{C}\left(\varphi_i^{(1)}\varphi_j^{(1)}\right)\qquad i=a,\,b \tag{3}$$

where a superscript "o" over a tensor indicates its symmetric traceless part. In Eq. (3), $\mathrm{C}\left(\varphi_i^{(1)}\right)$ and $\mathrm{C}\left(\varphi_i^{(1)}\varphi_j^{(1)}\right)$ are the linearized collision kernels whose explicit forms are also well known [6]-[10] and $\vec{d}_{ij}$ is the diffusive vector force given by

$$\vec{d}_{ab}=\nabla\frac{n_a}{n}+\frac{n_a n_b\left(m_a-m_b\right)}{n\rho}\frac{\nabla p}{p}-\frac{n_a n_b}{\rho p}\left(m_b e_a-m_a e_b\right)\cdot\vec{E}' \tag{4}$$

which satisfies the property $\vec{d}_{ij}=-\vec{d}_{ji}$. $\vec{E}'=\vec{E}+\vec{u}\times\vec{B}$ is the "effective" electric force, $\vec{u}$ the barycentric force defined as

$$\rho\vec{u}\left(\vec{r},t\right)=\sum_{i=a}^{b}\rho_i\vec{u}_i\left(\vec{r},t\right) \tag{5}$$

$\rho_i=m_i n_i$ $(i=a,\,b)$ and $\rho=\rho_a+\rho_b$. Also $\vec{u}_i\left(\vec{r},t\right)=\langle\vec{v}_i\rangle$ where $\langle\rangle$ is the standard average taken with $f_i\left(\vec{r},\vec{v}_i,t\right)$.

We must emphasize that, as has been recently shown [10], it is only in this representation that the Onsager reciprocity relations hold true, at least when $\vec{B}=0$. If $\vec{B}$ is different from zero the proof of such relations starting from the linearized integral equations for $\varphi_i^{(1)}$ will be discussed elsewhere. The solution to Eqs. (3) has been carefully outlined in Ref. [13]. Using Curie's theorem which allows setting $\nabla\vec{u}=0$, the solutions are found to be of the form

$$\varphi_i^{(1)}=-\vec{\mathbb{A}}_j\cdot\frac{\nabla T}{T}-\vec{\mathbb{D}}_i\cdot\mathbf{d}_{ij}\qquad i=a,\,b \tag{6}$$

where

$$\vec{\mathbb{A}}_i=\mathrm{A}_i^{(1)}\vec{c}_i+\mathrm{A}_i^{(2)}\left(\vec{c}_i\times\vec{B}\right)+\mathrm{A}_i^{(3)}\vec{B}\left(\vec{c}_i\cdot\vec{B}\right) \tag{7}$$

and a similar expression for $\vec{\mathbb{D}}_i$. The scalars $\mathrm{A}_i^{(j)}$ $(j=1,\,2,\,3)$ appearing in Eq. (7) are functions of of n, T, c_i^2, B^2 and $\left(\vec{c}_i\cdot\vec{B}\right)^2$.

When Eq. (6) and its analog for $\vec{\mathbb{D}}_i$. are substituted in Eq. (3) we get a set of linear integral equations for the unknown functions $\mathrm{A}_i^{(j)}$ and their analogs for the $\vec{\mathbb{D}}_i$ vector. As shown in Ref. [13], these equations are

$$f_i^{(0)}\left(\frac{m_i c_i^2}{2kT}-\frac{5}{2}\right)\vec{c}_i=f_i^{(0)}\left\{\mathrm{C}\left(\vec{c}_i\mathcal{R}_i\right)+\mathrm{C}\left(\vec{c}_i\mathcal{R}_i+\vec{c}_j\mathcal{R}_j\right)\right\} \tag{8}$$

where $\mathcal{R}_i = \mathrm{A}_i^{(1)} + B^2 \mathrm{A}_i^{(3)}$, $i = a$, b and

$$f_i^{(0)} \left(\frac{m_i c_i^2}{2kT} - \frac{5}{2} \right) \vec{c}_i = -f_i^{(0)} \frac{m_i}{\rho kT} iB \vec{c}_i \mathcal{G} - f_i^{(0)} \frac{e_i}{m_i} \vec{c}_i B \mathcal{A}_i + f_i^{(0)} \left\{ \mathrm{C} \left(\vec{c}_i \mathcal{A}_i \right) + \mathrm{C} \left(\vec{c}_i \mathcal{A}_i + \vec{c}_j \mathcal{A}_j \right) \right\} \tag{9}$$

In Eq. (9)

$$\mathcal{A}_i = \mathrm{A}_i^{(1)} + iB \mathrm{A}_i^{(2)} \tag{10}$$

$$\mathcal{G} = \mathrm{G}_B^{(1)} + iB \mathrm{G}_B^{(2)} \tag{11}$$

and

$$\mathrm{G}_B^{(k)} = \frac{1}{2} \sum_{j=a}^{b} e_j \int d\vec{c}_j f_j^{(0)} \mathrm{A}_j^{(k)} \left[c_j^2 - \frac{1}{B^2} \left(\vec{c}_j \cdot \vec{B} \right)^2 \right] \tag{12}$$

for $k = 1$, 2.

Similar results are obtained for the unknown functions $\mathrm{D}_i^{(j)}$ appearing in the analog of Eq. (7) for the vector $\vec{\mathbb{D}}_i$ but we shall not bother with them here since we will concentrate only in heat conduction in the plasma. The $\vec{\mathbb{D}}_i$vector is clearly related to diffusion phenomena [8]. Equations (8) and (9) are the basic ingredients required to discuss the problem of heat conduction. The curious reader will immediately realize the difference between these basic results and those used by Braginski in his paper on this subject [Ref. [5] pages 245-247]. In our method the unknown $\mathrm{A}_i^{(j)}$ functions satisfy integral equations, (8) and (9), where the collisional dynamics obeyed by the particles of the same and different species are contained in the linearized collision kernels about which no assumptions have yet been introduced. We shall now proceed in the next section to study the heat conduction in the dilute plasma.

3 Heat conduction in a fully ionized plasma

According to classical irreversible thermodynamics [1, 3, 14] the expression for the heat flux vector in a multicomponent system is given by

$$\vec{J}_q' = \vec{J}_q - \sum_i h_i \frac{\vec{J}_i}{m_i}$$

where $\vec{J}_i$ is the diffusion vector for species i and h_i its enthalpy. Since for an ideal gas $h_i = \frac{5}{2} kT$, where k is the Boltzmann constant, using the standard definition for $\vec{J}_q$, namely for our case

$$\vec{J}_q = \sum_{i=a}^{b} \frac{m_i}{2} \left\langle c_i^2 \right\rangle$$

we get that

$$\frac{1}{kT} \vec{J}_q' = \sum_{i=a}^{b} \int \left(\frac{m_i c_i^2}{2kT} - \frac{5}{2} \right) f_i \vec{c}_i d\vec{c}_i \tag{13}$$

since $\vec{J}_i = m_i \langle \vec{c}_i \rangle$. Also, $\vec{J}'_q = 0$ for $f_i = f_i^{(0)}$ so that, substituting Eq. (6) in Eq. (13) and ignoring diffusive contributions to $\vec{J}'_q$ we get that

$$(kT)^{-1} \vec{J}'_q = \sum_{i=a}^{b} \left(\frac{m_i c_i^2}{2kT} - \frac{5}{2} \right) f_i^{(0)} \varphi_i^{(1)} \vec{c}_i d\vec{c}_i \tag{14}$$

where $\varphi_i^{(1)}$ is now given by Eqs. (6) and (7).

To continue with the calculation of $\vec{J}'_q$ we now resort to the almost orthodox method in kinetic theory, namely to expand the unknown functions $\mathrm{A}_j^{(k)}$ in terms of a complete set of orthonormal functions, the Sonine polynomials, so that

$$\mathrm{A}_j^{(k)} = \sum_{m=0}^{\infty} a_j^{(k)(m)} S_{3/2}^{(m)} \left(c_i^2 \right) \qquad k = 1,\, 2,\, 3 \tag{15}$$

where the coefficients $a_j^{(k)(m)}$ are still to be determined from the integral equations (8) and (9) and thus depend on the interaction (Coulomb) potential between the species. Notice however that in spite of the complicated form for $\vec{\mathbb{A}}_i$ given in Eq. (7), when Eqs. (6) and (7) are introduced into Eq. (14) all integrals over $\vec{c}_i$ have the same structure, namely

$$\sum_{i=a}^{b} \left(\frac{m_i c_i^2}{2kT} - \frac{5}{2} \right) \frac{c_i^2}{3} f_i^{(0)} \sum_{m=0}^{\infty} a_i^{(k)(m)} S_{3/2}^{(m)} \left(c_i^2 \right) d\vec{c}_i = -\frac{5}{2} kT \sum_{i=a}^{b} \frac{n_i}{m_i} a_i^{(k)(m)} \qquad k = 1,\, 2,\, 3 \tag{16}$$

where use has been made of the well known property that [6]-[10]

$$\int_0^{\infty} e^{-x^2} S_n^{(p)}(x)\, S_n^{(q)}(x)\, x^{2n+1} dx = \frac{\Gamma(n+p+q)}{2p!} \delta_{pq} \tag{17}$$

and that $S_n^{(0)}(x) = 1$. Clearly then

$$\vec{J}'_q = -\frac{5}{2} k^2 T \sum_{i=a}^{b} \frac{n_i}{m_i} \left[a_i^{(1)(1)} \nabla T + a_i^{(2)(1)} \vec{B} \times \nabla T + a_i^{(3)(1)} \vec{B} \left(\vec{B} \cdot \nabla T \right) \right] \tag{18}$$

Equation (18) is an important result. Of the infinite number of coefficients required to specify the functions $\mathrm{A}_i^{(k)}$ appearing in Eq. (15) only one is required to compute the explicit form of the constitutive equation (18). This fact readily simplifies the solutions to the integral equations (8) and (9) as we shall see below. However, before doing so let us examine Eq. (18). If we take $\vec{B}$ in the direction of the z-axis, $\vec{B} \times \nabla T$ is a vector perpendicular to both $\vec{B}$ and ∇T whereas $\vec{B} \left(\vec{B} \cdot \nabla T \right) = B^2 \frac{\partial T}{\partial z} \vec{k}$ so that we may write that

$$\vec{J}'_q = -\frac{5}{2} k^2 T \sum_{i=a}^{b} \frac{n_i}{m_i} \left[\left(a_i^{(1)(1)} + B^2 a_i^{(3)(1)} \right) \nabla_{\parallel} T + a_i^{(1)(1)} \nabla_{\perp} T + B a_i^{(2)(1)} \nabla_s T \right] \tag{19}$$

where the last term represents a heat flow in the direction perpendicular to both $\vec{B}$ and ∇T. This is the well known Righi-Leduc effect [1]-[3]. Notice also that when $\vec{B}=0$

$$\vec{J}_q' = -\frac{5}{2}k^2 T \sum_{i=a}^{b} \frac{n_i}{m_i} a_i^{(1)(1)} \nabla T = -\kappa_{\parallel} \nabla T \tag{20}$$

which is the well known form for Fourier's heat conduction equation and the thermal conductivity is given by

$$\left(\kappa_{\parallel}\right)_{B=0} = \frac{5}{2}k^2 T \sum_{i=a}^{b} \frac{n_i}{m_i} a_i^{(1)(1)} \tag{21}$$

In Eq. (21) the coefficients $a_a^{(1)(1)}$ and $a_b^{(1)(1)}$ must arise from the solution to the integral equations (8) and (9) which indeed become identical to each other when $\vec{B}=0$. In its more general form, Eq. (19) thus reads as

$$\vec{J}_q' = -\kappa_{\parallel} \nabla_{\parallel} T - \kappa_{\perp} \nabla_{\perp} T - \kappa_s \nabla_s T \tag{22}$$

where the three conductivities are readily identified from Eq. (19). Once more a word of caution. Equation (22) appears to be the same as the one quoted by Braginski [see Eq. (4.33) of Ref.[5]] however, the evaluation of the coefficients $a_i^{(1)(m)}$ in our case radically differs from the procedure followed by this author. And worst, his definition of heat flux is foreign to the one used here, that is, not in agreement with irreversible thermodynamics.

To determine the $a_i^{(1)(m)}$ coefficients we need to solve Eqs. (8) and (9). The former one is straightforward and has been discussed in the literature, specially in Appendix B of Ref. [13]. The second one, however poses some problems due to the structure of its inhomogeneous term. Nevertheless by a subtle generalization of the procedure followed to solve Eq. (8), Eq. (9) is also solved using a variational procedure apparently due to Davison [15][16]. The steps are also outlined in the same Appendix B of Ref. [13]. We shall only quote the results here,

$$\begin{aligned} a_a^{(1)(0)} + B^2 a_a^{(3)(0)} &= 2.94\tau \\ a_a^{(1)(1)} + B^2 a_a^{(3)(1)} &= 1.96\tau \\ a_b^{(1)(1)} + B^2 a_b^{(3)(1)} &= \frac{0.058}{M_1}\tau \end{aligned} \tag{23}$$

also calling $a_i^{(m)} \equiv a_i^{(1)(m)} + iBa_i^{(2)(m)}$, $i=a,\, b,\, m=1,\,2$ following Eqs. (10) and (15) we get, after separating real and imaginary parts that

$$\mathrm{Re}\left[a_a^{(0)}\right] = a_a^{(1)(0)} = \left(56.3 - 662x^2 - 2.25x^4\right)\frac{\tau}{\Delta_1}$$

$$\mathrm{Im}\left[a_a^{(0)}\right] = Ba_a^{(2)(0)} = \left(647x + 2.2x^3\right)\frac{\tau}{\Delta_1}$$

$$\mathrm{Re}\left[a_a^{(1)}\right] = a_a^{(1)(1)} = \left(37.5 + 2147x^2 + 7.3x^4\right)\frac{\tau}{\Delta_1} \tag{24}$$

$$\mathrm{Im}\left[a_a^{(1)}\right] = Ba_a^{(1)(2)} = \left(206x + 2649x^3 + 9x^5\right)\frac{\tau}{\Delta_1}$$

$$\mathrm{Re}\left[a_b^{(1)}\right] = a_b^{(1)(1)} = \left(1.12 + 121.2x^2 + 154.4x^4\right)\frac{\tau}{M_1\Delta_1}$$

$$\mathrm{Im}\left[a_b^{(1)}\right] = Ba_b^{(1)(2)} = -\left(0.06x + 7x^3 + 9x^5\right)\frac{\tau}{M_1\Delta_1}$$

where

$$\Delta_1 = 19 + 2078x^2 + 2650x^4 + 9x^6 \tag{25}$$

and $x = \omega_e\tau = 1.76 \times 10^{11} B\tau$ where B is given in teslas. τ is the mean free time obtained from the only independent collision integral and is defined as[1]

$$\tau = \frac{4\left(2\pi\right)^{3/2}\sqrt{m_e}\left(kT\right)^{3/2}\epsilon_0^2}{ne^4\psi} \tag{26}$$

In Eq. (26) ψ is the so called logarithmic function which arises from using Debye's length as a cutoff length in the collision integrals, defined as

$$\psi = \ln\left[1 + \left(\frac{16\pi kT\lambda_D\epsilon_0}{e^2}\right)^2\right] \tag{27}$$

where λ_D is the Debye's length, namely

$$\lambda_D = \sqrt{\frac{\epsilon_0 kT}{ne^2}} \tag{28}$$

Notice that the consistency requirement that $\left(a_a^{(1)(1)}\right)_{B=0} = \left(\mathrm{Re}\left[a_a^{(1)}\right]\right)_{B=0}$ is satisfied within the approximation used here.
Therefore, summarizing the three thermal conductivities in Eq. (22) are given by

$$\kappa_\parallel = \frac{5}{4}\frac{k^2T}{m_e} \times 2.01n\tau \tag{29}$$

$$\kappa_\perp = \frac{5}{4}\frac{k^2T}{m_e}\frac{n\tau}{\Delta_1} \times \left(38.7 + 2270x^2 + 161x^4\right) \tag{30}$$

$$\kappa_s = \frac{5}{4}\frac{k^2T}{m_e}\frac{n\tau}{\Delta_1} \times \left(206x + 2644x^3\right) \tag{31}$$

where ψ is defined in Eq. (27).

We emphasize that Eqs. (29)-(31) are valid for a fully ionized hydrogen plasma ($n_a = n_b = n/2$, $m_b \gg m_a = m_e$) in a first order in the gradients approximation, namely the Navier-Stokes-Fourier regime. These are the main results of this paper.

[1] The numerical factors appearing in the six polynomials quoted in Eqs. (24) substitute those published in Appendix B of Ref. [13] which result from a revised calculation.

4 Discussion of the results

The first thought that a reader may have concerning the nature of Eqs. (29)-(31) is to ask how they compare to those obtained from the well accepted and dominant calculations obtained much earlier by Spitzer [17] and by Braginski [5]. As already pointed out in full detail by Balescu in Ref. [6] and emphasized by us in a recent paper [13] this is quite difficult. Neither of both authors, besides using the Fokker-Planck and Landau kinetic equations, respectively, performed their calculations within the framework of classical irreversible thermodynamics. In particular, the correct definition for the heat flux in a multicomponent mixture, e. g. Eq. (12), was ignored. Therefore the expressions that they quote for the thermal conductivities are not equivalent to ours.

The most interesting feature of our results is that the three thermal conductivities, exhibit a behavior which is similar to that shown by the results obtained by Balescu who used a Landau type kinetic equation which he solved using the "moments" method. This result is gratifying. It shows indeed that using the full Boltzmann equation and solving the ensuing integral equations which define the coefficients appearing in the explicit results for the transport coefficients leads to results that approximately equal to those obtained from the Landau equation when the practically "exact" 21 moment approximation is used. Comparing figure 1 of our paper with figure 5.1 in Balescu's book confirms this statement. Thus, his prediction that different methods used to compute transport coefficients will yield results that will be within a 10% difference from each other turns out to be sustained.

Another issue is important. Working with the moment method leads to results which are not clearly related to the order of the macroscopic gradients in the system. This was first pointed out by Grad in his monumental work on this subject [19]. Referring to the quantity that appears in his case namely, the one resulting from the collision integral in the thirteen moment solution to the Boltzmann equation, he asserts that "such a parameter has a number of interpretations" (see p. 271-72 of Ref. [19]) . Further, if one wants to classify the resulting equations in terms of power in the gradients one must apply the Chapman-Enskog expansion not to the Boltzmann equation but to the moment equations. It is only then, as has been extensively discussed in Ref. [20] one may extract the Euler, Navier-Stokes-Fourier, Burnett and higher order in the gradients contributions. Therefore Balescu's nearly exact results as he claims, obtained with the 21 moment method, are not clearly related to the ordinary hydrodynamic hierarchy of equations. We believe this is the main reason why our results are somewhat different from his. It is outside the scope of this paper to attempt a detailed comparison of both methods, mainly a purely algebraic issue.

Finally we wish to remark also that these results together with similar ones here obtained for the Dufour coefficient may be useful in accounting for dissipative phenomena which are becoming rather important in the physics of the intracluster medium [21]-[23].

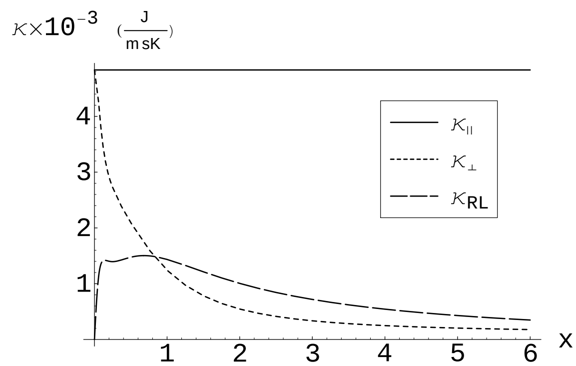

Fig. 1. The three thermal conductivities as functions of $x = \omega_e \tau$ for $T = 10^6 K$ and $n = 10^{15} m^{-3}$.

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